

Electroweak Parameters in the \overline{MS} -Scheme

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Abstract

We study electroweak parameters sensitive to the radiative corrections, such as ρ – ratio and $H \rightarrow f\bar{f}$ decay rates in the \overline{MS} -scheme in the heavy t -quark mass limit $m_t \gg m_w$. In \overline{MS} -scheme the two-loop electroweak corrections $\sim m_t^4$ dominate over the QCD corrections $\sim \alpha_s m_t^2$ for ρ . The relation between the *on-shell* coupling constants and \overline{MS} -parameters is found to be rather sensitive to the Higgs boson mass.

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Introduction

The standard model of electroweak interactions is a renormalizable quantum field theory. Thus one has the possibility of “precision tests” of the theory, and sensitivity to “new physics”. Two important issues are the scheme dependence of the necessarily truncated perturbation series, and the decoupling or nondecoupling of heavy masses from low energy processes — in particular, the dependence on the top and Higgs masses (m_t, m_H).

The renormalization of the complete standard electroweak theory is quite complicated [6]. There are all the complications of quantizing a nonabelian gauge theory, as well as mass generation by spontaneous symmetry breaking, $\gamma - Z^0$ mixing, etc. It is useful to find a substantially simpler framework in which the issues raised above can be studied.

The top and Higgs masses enter the complete standard electroweak theory through the Yukawa coupling of the top and the Higgs and the quartic scalar self-coupling and the vev of the (unshifted) higgs field.

$$\mathcal{Y}_t \sim \frac{m_t}{v}, \quad \lambda \sim \frac{m_H^2}{v^2}$$

The simplified framework we will study is the “gaugeless limit” in which the electroweak gauge coupling constants are set to zero

$$g_2, g_1 \rightarrow 0$$

and

$$M_W, M_Z \sim gv \rightarrow 0$$

Also, all Yukawa coupling constants except \mathcal{Y}_t are set to 0

$$\mathcal{Y}_f \rightarrow 0 \quad (f \neq t)$$

Keeping only the heavy quark generation, we study the scalar - (heavy) quark sector, consisting of a physical Higgs, three unphysical Goldstone bosons and the t and b quarks. This limit has no gauge degrees of freedom, gauge fixing, or Fadeev- Popov factor. It has a global $SU(2)_L$ symmetry and spontaneous symmetry breaking with three massless Goldstone bosons. (This treatment has substantial overlap with, but is not identical to, considerations based on the “Equivalence Theorem” [1].)

In this much simpler model, we will study in some detail the issues raised in the first paragraph. In the end, to make contact with observed low energy processes, we will find that we can not completely avoid the complications of the renormalization of the full theory.

1 Renormalization

The fields of the reduced theory occur as singlets and doublets under the global $SU(2)_L$.

$$\Psi_l = \begin{pmatrix} \mathcal{T} \\ \mathcal{B} \end{pmatrix}_L, \quad \mathcal{T}_R, \quad \mathcal{B}_R, \quad \begin{pmatrix} \Phi_+ \\ \Phi_0 \end{pmatrix} \quad (1)$$

$$\Phi_+ = \varphi_+ \equiv \varphi = \frac{\varphi_1 - i\varphi_2}{\sqrt{2}}, \quad \Phi_0 = \frac{\mathcal{H} - i\varphi_0}{\sqrt{2}}$$

$$\tilde{\Phi} = i\tau_2 \Phi^\dagger = \begin{pmatrix} \bar{\Phi}_0 \\ -\Phi_- \end{pmatrix}$$

The Lagrangian is

$$\begin{aligned} \mathcal{L} = & \bar{\Psi}_L i\gamma \cdot \partial \Psi_L + \bar{\mathcal{T}}_R i\gamma \cdot \partial \mathcal{T}_R + \bar{\mathcal{B}}_R i\gamma \cdot \partial \mathcal{B}_R \\ & + \partial \Phi^\dagger \partial \Phi - \mu_0^2 \Phi^\dagger \Phi - \lambda_0 (\Phi^\dagger \Phi)^2 \\ & - \mathcal{Y}_{0t} (\bar{\Psi}_L \tilde{\Phi} \mathcal{T}_R + h.c.) \end{aligned} \quad (2)$$

The fields are unrenormalized canonical fields, and μ_0^2, λ_0 , and \mathcal{Y}_{0t} are bare mass and coupling constants .

To implement perturbation theory in the broken symmetry phase ($\mu_0^2 < 0, \langle \mathcal{H} \rangle \neq 0$), the Higgs field is shifted

$$\langle \mathcal{H} \rangle = \mathcal{V}, \quad \mathcal{H} = \mathcal{V} + \hat{\mathcal{H}} \quad (3)$$

The condition that one is perturbing about the correct vacuum is

$$\langle \hat{\mathcal{H}} \rangle = 0 \quad (4)$$

This condition determines \mathcal{V} as a function of the parameters of the theory.

The bare fields and parameters of (1),(2) are reexpressed as renormalized fields and parameters, multiplied by appropriate Z -factors.

$$\begin{aligned} \mu_0^2 &= Z_\mu \mu^2, & \lambda_0 &= Z_\lambda \lambda, & \mathcal{Y}_{0t} &= Z_y \mathcal{Y}_t \\ \Psi_L &= \sqrt{Z_L} \psi_L, & \mathcal{T}_R &= \sqrt{Z_t} t_R, & \mathcal{B}_R &= \sqrt{Z_b} b_R \\ \{\hat{\mathcal{H}}, \varphi_0, \varphi, \varphi^\dagger\} &= \sqrt{Z_\phi} \{h, \phi_0, \phi, \phi^\dagger\}, & \mathcal{V} &= \sqrt{Z_\phi} V \end{aligned} \quad (5)$$

In terms of these renormalization constant , the Yukawa and quartic scalar vertex renormalizations are

$$\sqrt{Z_L} \sqrt{Z_t} \sqrt{Z_\phi} Z_y = Z_3, \quad Z_\phi^2 Z_\lambda = Z_4 \quad (6)$$

Note that we have introduced a common field strength renormalization constant for all four of the scalar fields. This is in accord with the dictat to introduce only counter terms which respect the symmetries of the original “bare” Lagrangian, in this case, the $O(4)$ symmetry of the purely bosonic part of the Lagrangian (2). Then after spontaneous symmetry breaking, the currents associated with the $O(4)$ generators will still be conserved. The fact that there is no longer a common mass shell will have to be taken into account in the LSZ reduction formulas relating Green functions to S-Matrix elements (see section 3).

When (3) and (5) are substituted into (2), and all Z ’s are rewritten as $Z = 1 + \delta Z$, the Lagrangian (2) is rewritten as a lengthy sum of terms, starting with a sum of terms of the same form as (2) but with all bare fields and parameters replaced by renormalized ones,

plus terms proportional to V (and containing no δZ), plus counter terms proportional to one or more δZ 's. The procedure is quite standard, and we will only write out individual pieces as we need to discuss them.

The first order of business is to implement the stability condition (4) To do this, compute all tadpole graphs, including counter terms , to a given order (number of loops) in perturbation theory , and expand the renormalized V

$$\begin{aligned} V &\equiv \zeta_v v , \\ \zeta_v &= 1 + \zeta_v^{(1)} + \zeta_v^{(2)} + \dots \end{aligned} \quad (7)$$

and adjust ζ_v (equivalently V) to satisfy (4) order by order. The terms in \mathcal{L} , linear in the shifted Higgs field, are

$$\begin{aligned} \mathcal{L}_1 &= -\mathcal{V} \hat{\mathcal{H}}(\mu_0^2 + \lambda_0 \mathcal{V}^2) = -V h (Z_\phi Z_\mu \mu^2 + Z_\phi^2 Z_\lambda \lambda V^2) \\ &= -v h \{(\mu^2 + \lambda v^2) + (\zeta_v - 1)(\mu^2 + \lambda v^2) + [(Z_\phi Z_\mu - 1)\mu^2 + (Z_4 \zeta_v^2 - 1)\lambda v^2]\} \\ &+ \text{two-loop counterterms} \end{aligned} \quad (8)$$

Zeroth order fixes

$$v^2 = \frac{-\mu^2}{\lambda}, \quad (\mu^2 = -\frac{M^2}{2} < 0) \quad (9)$$

and the one-loop counter term may be written as

$$\begin{aligned} \mathcal{L}_1 &= v \delta \mu^2 h \\ \delta \mu^2 &= -(Z_\phi Z_\mu - Z_4 \zeta_v^2) \mu^2 = (Z_\phi Z_\mu - Z_4 \zeta_v^2) \lambda v^2. \end{aligned} \quad (10)$$

Computation of $\langle h \rangle$ in one-loop order (fig. 1) gives

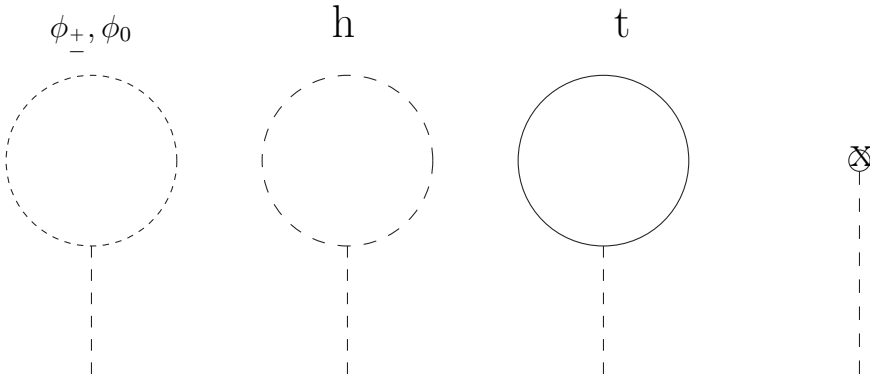


Fig.1. Tad-pole diagrams. Long dashes - Higgs, short dashes - Goldstone, solid - top. Big dot with x is counter term linear in Higgs field.

$$\langle h \rangle = \frac{-1}{M^2} \left[2\lambda v A_0 + \lambda v A_0 + 3\lambda v A_M - 2\sqrt{2} N_c \mathcal{Y} m A_m - v \delta \mu^2 \right] \quad (11)$$

where the regularized Feynman integral is

$$A_M = i \int_{reg} (dp) \frac{1}{p^2 - M^2 + i\epsilon} \quad (12)$$

(In the reduced theory there are only two masses, $m_t = m$ and $m_H = M$. The Goldstone bosons remain massless). Expanding (10) to one-loop order, requiring (11) to be zero, fixes $(\sqrt{2}m = \mathcal{Y} v)$

$$\delta\mu^2 = \frac{M^2}{2} (\delta Z_\phi + \delta Z_\mu - \delta Z_4 - 2\delta\zeta_v) = 3\lambda(A_0 + A_M) - 2N_c y^2 A_m \quad (13)$$

We will subsequently fix δZ_ϕ as the Goldstone boson field renormalization constant and δZ_4 as the Higgs boson mass renormalization constant ($M^2 = 2\lambda v^2$). This leaves δZ_μ and $\delta\zeta_v$ to be adjusted to satisfy (13). At this point we find a significant difference between momentum dependent (“*MOM*”) renormalization schemes, and momentum independent, in particular \bar{MS} , renormalization schemes. In an *MOM* scheme, δZ_μ is unrestricted so we can choose $\delta\zeta_v$ to be zero and adjust δZ_μ to satisfy (13). In \bar{MS} , δZ_μ is restricted to a divergent part; no adjustable finite part is admitted. Thus when we compute the integrals on the right hand side of (13) with dimensional regularization the $\frac{1}{\epsilon}$ terms are matched by $\frac{1}{\epsilon}$ terms from the δZ 's on the left hand side, but there is a finite part remaining which determines $\delta\zeta_v$. In dimensional regularization

$$\begin{aligned} A_M &= \frac{M^2}{16\pi^2} \left[-\Delta_\epsilon + \ln\left(\frac{M^2}{\mu^2}\right) - 1 \right] \\ \Delta_\epsilon &= \frac{2}{4-d} - \gamma_E + \ln(4\pi) \end{aligned} \quad (14)$$

μ^2 is the arbitrary scale introduced in dimensional regularization, not to be confused with the arbitrary renormalized mass parameter in (5), (9). Then

$$\delta\bar{\zeta}_v = \frac{1}{16\pi^2} \left[3\lambda \left(1 - \ln\left(\frac{M^2}{\mu^2}\right) \right) - 2N_c \mathcal{Y}^2 \frac{m^2}{M^2} \left(1 - \ln\left(\frac{m^2}{\mu^2}\right) \right) \right] \quad (15)$$

Note that the right hand side of (13) depends on the regularization of the divergent integrals, but makes no reference to the renormalization scheme. The combination of renormalization constant plus the one-loop shift in the v.e.v. occuring in $\delta\mu^2$ is scheme independent.

The terms in the Lagrangian, quadratic in the scalar fields are

$$\begin{aligned} \mathcal{L}_2 &= \partial\Phi^\dagger\partial\Phi - \frac{1}{2}(\mu_0^2 + \lambda_0\mathcal{V}^2)(2\varphi^\dagger\varphi + \varphi_0^2) - \frac{1}{2}(\mu_0^2 + 3\lambda_0\mathcal{V}^2)h^2 \\ &= \partial\Phi^\dagger\partial\Phi + \frac{\delta\mu^2}{2}(2\phi^\dagger\phi + \phi_0^2) - \frac{1}{2}(2\lambda v^2)h^2 + \frac{1}{2}(\delta\mu^2 - (Z_4\zeta_v^2 - 1)(2\lambda v^2))h^2 \end{aligned} \quad (16)$$

$$M_\phi = 0, \quad M^2 = 2\lambda v^2 \quad (17)$$

The one-loop counter terms (for $-i\Sigma$) generated by (16) are

$$\phi : i(Z_\phi - 1)q^2 + i\delta\mu^2 \quad (18)$$

$$h : i(Z_\phi - 1)q^2 + i\delta\mu^2 - i(Z_4\zeta_v^2 - 1)M^2 \quad (19)$$

The one-loop Goldstone boson self-energy diagrams, including tadpoles and counter terms are shown in fig. 2.

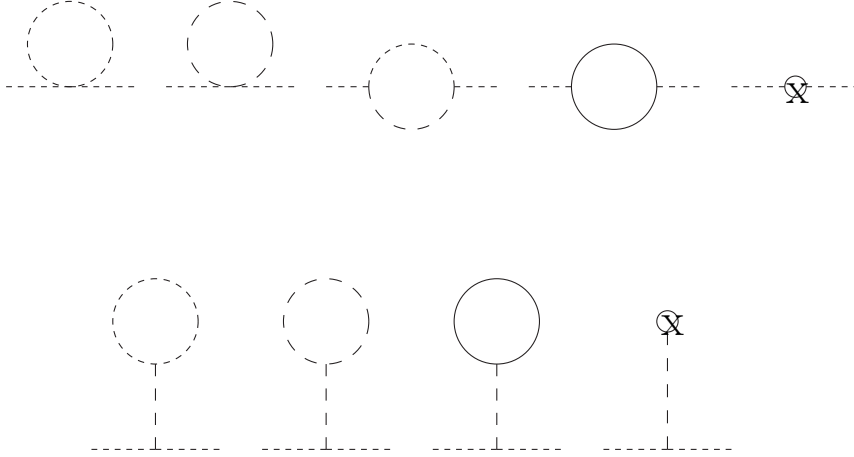


Fig.2. The one-loop self-energy diagrams of the Goldstone field.

The tadpole diagrams plus the tadpole counter term in the second line of fig. 2 add to zero by our previous choice of $\delta\mu^2$ (13). The contribution of the Feynman diagrams and counter term of the first line of fig. 2 to the renormalized self-energy function is, for Σ_+

$$\begin{aligned} -i\Sigma_{\phi_+} = & -i2\lambda[2A_0 + \frac{1}{2}A_0 + \frac{1}{2}A_M + 2\lambda v^2 I_{M0}(q^2)] \\ & + i\mathcal{Y}^2 N_c[A_0(q^2) + A_m(0) - (q^2 - m^2)I_{m0}(q^2)] + i\delta\mu^2 + i\delta Z_\phi q^2 \end{aligned} \quad (20)$$

and for Σ_0

$$\begin{aligned} -i\Sigma_{\phi_0} = & -i2\lambda[A_0 + \frac{3}{2}A_0 + \frac{1}{2}A_M + 2\lambda v^2 I_{M0}(q^2)] \\ & + i\mathcal{Y}^2 N_c[A_m(q^2) + A_m(0) - q^2 I_{mm}(q^2)] + i\delta\mu^2 + i\delta Z_\phi q^2 \end{aligned} \quad (21)$$

Here

$$A_m(q^2) = i \int_{reg} (dl) \frac{1}{(l - q)^2 - m^2}, \quad I_{ab}(q^2) = i \int_{reg} (dl) \frac{1}{(l^2 - a^2)((l - q)^2 - b^2)} \quad (22)$$

and

$$A_m(0) = A_0(0) + m^2 I_{m0}(0) \quad (23)$$

for any regularization. Using (23) and (13) in (20), (21), we find

$$\Sigma_{\phi_+}(0) = \Sigma_{\phi_0}(0) = 0 \quad (24)$$

i.e. the same $\delta\mu^2$ which enforces $\langle h \rangle = 0$ also makes $M_\phi = 0$ for both charged and neutral Goldstone bosons. This is the reason that it is possible to choose $\delta\zeta_v$ equal to zero in an MOM scheme. It is interesting to add the two counter term contributions from the first and second lines of fig. 2.

$$i\delta\mu^2 + (-2i\lambda v) \frac{i}{-M^2} iv \delta\mu^2 = i\delta\mu^2 \left(1 - \frac{2\lambda v^2}{M^2}\right) = 0$$

Thus, the sum of all the Feynman diagrams, including the tadpole contributions, but no counter terms also gives zero for $q^2 = 0$. This is a consistency check, that the Goldstone theorem is satisfied independently of renormalization scheme.

The δZ_ϕ counter term is determined by the terms linear in q^2 in (20), (21)

$$\Sigma'_{\phi_+}(0) = 2\lambda[M^2 I'_{M0}(0)] - \mathcal{Y}^2 N_c[A'_0(0) - I_{m0}(0) + m^2 I'_{m0}(0)] - \delta Z_\phi \quad (25)$$

$$\Sigma'_{\phi_0}(0) = 2\lambda[M^2 I'_{M0}(0)] - \mathcal{Y}^2 N_c[A'_m(0) - I_{mm}(0)] - \delta Z_\phi \quad (26)$$

In an on-shell *MOM* renormalization scheme, the field strength renormalization constant δZ_ϕ is fixed such that the residue of the pole of the renormalized ϕ propagator is unity. In the spontaneously broken symmetry phase, this would require separate δZ_{ϕ_+} and δZ_{ϕ_0} . But then the renormalization reparametrization (5) of the Lagrangian would introduce explicitly $SU(2)_L$ breaking terms into the Lagrangian (2). So we stick with a single δZ_ϕ chosen, for the moment arbitrarily, to be δZ_{ϕ_+} . However, δZ_{ϕ_0} will also be required for later use in the LSZ reduction formulas. If the integrals in (25),(26) are computed with dimensional regularization, the results are

$$\delta Z_\phi \equiv \delta Z_{\phi_+} = \frac{\lambda}{16\pi^2}[-1] + \frac{\mathcal{Y}^2}{16\pi^2} N_c[-\Delta_\epsilon + \ln(\frac{m^2}{\mu^2}) - \frac{1}{2}] \quad (27)$$

$$\delta Z_{\phi_0} = \frac{\lambda}{16\pi^2}[-1] + \frac{\mathcal{Y}^2}{16\pi^2} N_c[-\Delta_\epsilon + \ln(\frac{m^2}{\mu^2})] \quad (28)$$

We remark here that in a treatment of the entire electroweak theory, δZ_ϕ may be fixed by a Renormalization condition in the gauge sector. We will return to this point when we discuss a Ward-Slavnov-Taylor identity of the theory. The \overline{MS} field strength renormalization is the same for ϕ_+ and ϕ_0 . It is simply the common Δ_ϵ contribution.

$$\delta \bar{Z}_\phi = N_c \frac{\mathcal{Y}^2}{16\pi^2} [-\Delta_\epsilon] \quad (29)$$

For the physical Higgs propagator, there is a set of self-energy Feynman diagrams corresponding to those of fig. 2 for the Goldstone bosons . The resulting renormalized self-energy function is

$$\begin{aligned}
-i\Sigma_h(q^2) = & -2i\lambda \left[\frac{3}{2}A_0 + \frac{3}{2}A_M + 2\lambda v^2 \left(\frac{3}{2}I_{00}(q^2) + \frac{9}{2}I_{MM}(q^2) \right) \right] \\
& + i\mathcal{Y}^2 N_c \left[A_m(q^2) + A_m(0) - (q^2 - 4m^2)I_{mm}(q^2) \right] \\
& + i\delta Z_\phi q^2 + i\delta\mu^2 - i(Z_4\zeta_v^2 - 1)M^2
\end{aligned} \tag{30}$$

The on-shell mass renormalization condition for an unstable particle is generally taken to be $\text{Re}\Sigma(M^2) = 0$. Substitute $\delta\mu^2$ from (13)(recall that it is scheme independent) into (30), and set $q^2 = M^2$. (With dimensional regularization $A_m(q^2) = A_m(0) \equiv A_m$).

$$\begin{aligned}
\Sigma_h(M^2) = & 3\lambda M^2 \left[I_{00}(M^2) + 3I_{MM}(M^2) \right] \\
& + N_c \mathcal{Y}^2 \left[(M^2 - 4m^2)I_{mm}(M^2) \right] - \delta Z_\phi M^2 + 2\delta\zeta_v M^2 + \delta Z_4 M^2
\end{aligned} \tag{31}$$

Since δZ_ϕ and $\delta\zeta_v$ are previously fixed, in either MOM or \bar{MS} , taking the real part of $I_{00}(M^2)$ (and $I_{mm}(M^2)$ if $M > 2m$) and setting $\text{Re}\Sigma_h(M^2)$ to zero fixes δZ_4 in MOM , while just matching Δ_ϵ terms fixes δZ_4 in \bar{MS} .

Taking the derivative of (30) gives

$$\begin{aligned}
\Sigma'_h(M^2) = & 3\lambda M^2 [I'_{00}(M^2) + 3I'_{MM}(M^2)] \\
-N_c \mathcal{Y}^2 \left[-I_{mm}(M^2) - (M^2 - 4m^2)I'_{mm}(M^2) \right] - \delta Z_\phi.
\end{aligned} \tag{32}$$

The fact that this is not zero for δZ_ϕ of (27) has implications for calculation of processes in which a physical Higgs particle appears as an external line. When $\Sigma'_h(M^2)$ is not zero, virtual radiative corrections to external Higgs lines survive LSZ amputation. These contributions can be determined from the difference $\delta Z_h - \delta Z_\phi$. This is simple in the limit $M^2 \gg m^2$ ($\lambda \gg \mathcal{Y}^2$).

$$\delta Z_h = \frac{\lambda}{16\pi^2} \left[3 + 9 \left(1 - \frac{2\pi}{3\sqrt{3}} \right) + \mathcal{O}\left(\frac{m^2}{M^2}\right) \right] \tag{33}$$

Comparison with (27) gives

$$\delta Z_h - \delta Z_\phi = \frac{\lambda}{16\pi^2} \left[13 - 2\sqrt{3}\pi + \mathcal{O}\left(\frac{m^2}{M^2}\right) \right] \tag{34}$$

From consideration of the bosonic one- and two- point functions we have fixed the one-loop bosonic renormalization constants , $\delta Z_\phi, \delta Z_\mu, \delta Z_4$, and the one-loop shift of the vev, $\delta\zeta_v$, in both MOM and \bar{MS} schemes. There remain bosonic three- and four-point functions with ultraviolet divergences, but a sequence of Ward identities (e.g. [2]) guarantee that these will be rendered finite by the counter terms generated in (2) by (3) and (5), (6).

We turn now to the relation between the bosonic MOM mass and coupling constant (M^*, λ^*) and the $\bar{M}\bar{S}$ mass and coupling constant $(\bar{M}, \bar{\lambda})$. The OS mass is determined by the mass shell condition

$$0 = \text{Re}D_h^{*-1}(M^{*2}) = (M^{*2}) - (M^{*2}) - \text{Re}\Sigma_h^*(M^{*2}) \quad (35)$$

This fixes δZ_4^* in (31). The bosonic MOM parameters are M^* and λ^* . M^* is the physical Higgs mass (modulo the usual problems of unstable particles). Since δZ_4^* is fixed by the mass condition, it is not available to define the coupling constant as the value of a vertex function at some kinematic point. λ^* is traded for v^* which is to be determined in terms of the accurately known low energy electroweak parameters G_F, α, M_Z . Thus

$$\lambda^* = \frac{M^{*2}}{2v^{*2}} \quad (36)$$

In $\bar{M}\bar{S}$ Eqn. (35) becomes

$$0 = \text{Re}\bar{D}_h^{-1}(M^{*2}) = M^{*2} - \bar{M}^2 - \text{Re}\bar{\Sigma}_h(M^{*2}) \quad (37)$$

Then

$$\begin{aligned} \frac{M^{*2}}{\bar{M}^2} &= 1 + \text{Re}\bar{\Sigma}_h(M^{*2})/\bar{M}^2 \\ &= 1 + 3\lambda[\text{Re}I_{00}(M^2) + 3I_{MM}(M^2)] \\ &+ N_c\mathcal{Y}^2[(1 - 4\frac{m^2}{M^2})\text{Re}I_{mm}(M^2)] - \delta\bar{Z}_\phi + \delta\bar{Z}_4 + 2\delta\bar{\zeta}_v \end{aligned} \quad (38)$$

due to (31). $\delta\bar{Z}_\phi$ and $\delta\bar{Z}_4$ just remove the Δ_ϵ terms. Then

$$\frac{M^{*2}}{\bar{M}^2} = 1 + \frac{\lambda}{16\pi^2}[12\ln(\frac{M^2}{\mu^2}) - 24 + 3\sqrt{3}\pi + \mathcal{O}(\frac{m^2}{M^2})] + 2\delta\bar{\zeta}_v \quad (39)$$

The $\bar{M}\bar{S}$ mass depends on the $\bar{M}\bar{S}$ one-loop shift of the v.e.v., $\delta\bar{\zeta}_v$, given in (15). Note that an alternative definition of the $\bar{M}\bar{S}$ mass is possible. If (39) is multiplied through by \bar{M}^2 , the collected terms $\bar{M}^2(1 + 2\delta\bar{\zeta}_v)$ are just the one-loop expansion of $2\lambda V^2$ i.e the mass squared is defined as the coupling constant times the exact vev squared, rather than the tree level vev squared, and the $\delta\bar{\zeta}_v$ in (39) is absorbed in that redefinition of the $\bar{M}\bar{S}$ mass. However, that definition of the $\bar{M}\bar{S}$ mass is generally gauge dependent when the gauge sector is included. For the quartic coupling constant we have

$$\begin{aligned} \frac{\lambda^*}{\bar{\lambda}} &= \frac{M^{*2}\bar{v}^2}{\bar{M}^2 v^{*2}} = \frac{M^{*2}\zeta_v^{*2}Z_\phi^*}{\bar{M}^2\bar{\zeta}_v^2\bar{Z}_\phi} = \frac{M^{*2}}{\bar{M}^2} [1 + (\delta Z_\phi^* - d\bar{Z}_\phi) - 2\delta\bar{\zeta}_v] \\ &= 1 + \frac{\lambda}{16\pi^2}[12\ln(\frac{M^2}{\mu^2}) - 25 + 3\sqrt{3}\pi + \mathcal{O}(\frac{m^2}{M^2})] \end{aligned} \quad (40)$$

with δZ_ϕ from (27). Note the cancellation of $\delta\zeta_v$. The ratio $\lambda^*/\bar{\lambda}$ is independent of the choice of $\delta\zeta_v^*$ discussed below (13). An alternative calculation, which is manifestly independent of $\delta\zeta_v$, is

$$\frac{\lambda^*}{\bar{\lambda}} = \frac{\bar{Z}_\lambda}{Z_\lambda^*} = \frac{\bar{Z}_4 Z_\phi^{*2}}{Z_4^* \bar{Z}_\phi^2} = 1 + (\delta\bar{Z}_4 - \delta Z_4^*) + 2(\delta Z_\phi^* - \delta\bar{Z}_\phi) \quad (41)$$

Again, keeping just the leading terms for $M^2 \gg m^2$, eqs (31), (27) and $\delta\zeta_v^* = 0$, give

$$\begin{aligned} \delta Z_4^* &= -3\lambda [\text{Re}I_{00}(M^2) + 3I_{MM}(M^2)] + \frac{\lambda}{16\pi^2}[-1] \\ &= \frac{\lambda}{16\pi^2} \left[-3 \left(-\Delta_\epsilon + \ln\left(\frac{M^2}{\mu^2}\right) - 2 \right) - 9 \left(-\Delta_\epsilon + \ln\left(\frac{M^2}{\mu^2}\right) - 2 + \frac{\pi}{\sqrt{3}} \right) - 1 \right] \end{aligned} \quad (42)$$

Subtracting $\delta\bar{Z}_4$ just removes the Δ_ϵ 's. Collecting the pieces in (41) reproduces the result (40) [3].

Proceeding to the fermion Green functions and renormalization conditions, the terms in the Lagrangian (2) quadratic in the fermion fields are

$$\begin{aligned} \mathcal{L}_2 &= \bar{T}i\gamma \cdot \partial T + \bar{B}i\gamma \cdot \partial B - \mathcal{Y}_{0t} \frac{\mathcal{V}}{\sqrt{2}} \bar{T}T \\ &= \bar{t}(i\gamma \cdot \partial - m)t + \bar{b}i\gamma \cdot \partial b - (Z_3\zeta_v - 1)m\bar{t}t \\ &\quad + \delta Z_L \left(\bar{t}_L i\gamma \cdot \partial t_L + \bar{b}_L i\gamma \cdot \partial b_L \right) \\ &\quad + \delta Z_t \bar{t}_R i\gamma \cdot \partial t_R + \delta Z_b \bar{b}_R i\gamma \cdot \partial b_R \end{aligned} \quad (43)$$

Since we have only one nonzero Yukawa coupling constant and fermion mass, we usually suppress the subscript t : $m_t \equiv m = \mathcal{Y} \frac{v}{\sqrt{2}}$, $m_b = 0$. This generates the fermion two-point counter terms (for $-i\Sigma$):

$$t : \quad i\delta Z_L \gamma p \frac{1-\gamma_5}{2} + i\delta Z_t \gamma p \frac{1+\gamma_5}{2} - i(Z_3\zeta_v - 1)m \quad (44)$$

$$b : \quad i\delta Z_L \gamma p \frac{1-\gamma_5}{2} + i\delta Z_b \gamma p \frac{1+\gamma_5}{2} \quad (45)$$

The vertex counter terms generated by substituting (5), (6) into the trilinear terms in (2) are displayed in fig. 3.

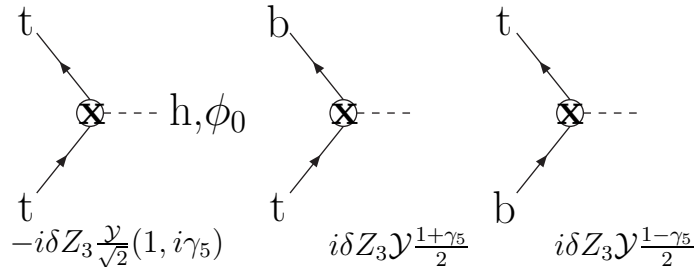


Fig.3. Yukawa vertex counter terms.

The inverse of the complete renormalized propagator is

$$S^{-1}(p) = \gamma p - m - \Sigma(p) = \gamma p(1 - A - B\gamma_5) - m(1 - C) \quad (46)$$

Fig. 4 shows the one-loop Feynman diagrams and the counter term contributing to Σ :

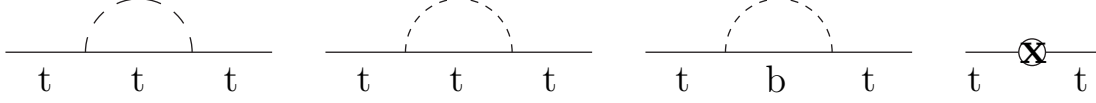


Fig.4. The one-loop self-energy diagrams of the t quark.

$$\Sigma = \Sigma_{FD} + \Sigma_{ct}:$$

$$\begin{aligned} A &= a - \frac{1}{2} (\delta Z_L + \delta Z_t), \\ B &= b - \frac{1}{2} (-\delta Z_L + \delta Z_t), \\ C &= c - (Z_3 \zeta_v - 1) \end{aligned} \quad (47)$$

with a, b, c coming from the Feynman diagrams Σ_{FD} . Rationalizing one obtains

$$S(p) = \frac{\gamma p(1 - A - B\gamma_5) + m(1 - C)}{p^2((1 - A)^2 - B^2) - m^2(1 - C)^2} \quad (48)$$

The mass shell condition for $p^2 = m^{*2}$ is

$$\begin{aligned} 0 &= m^{*2}((1 - A)^2 - B^2) - m^2(1 - C)^2 \\ &= m^{*2}(1 - 2A) - m^2(1 - 2C) + \text{two-loop} \end{aligned} \quad (49)$$

The on-shell MOM renormalization condition is then ($m^2 = m^{*2}$)

$$A^* - C^* = 0 \quad (50)$$

The electroweak contribution to $\frac{m^{*2}}{m^2}$ is

$$\frac{m^{*2}}{m^2} = 1 + 2(\bar{A} - \bar{C}) = 1 + 2(\bar{a} - \bar{c}) + 2\delta\bar{\zeta}_v \quad (51)$$

where \bar{a}, \bar{c} are just a, c with the Δ_ϵ 's removed. The result of that calculation from the Feynman diagrams of fig. 4 is [15]

$$\begin{aligned} 2(\bar{a} - \bar{c}) &= \frac{\mathcal{Y}^2}{16\pi^2} \left[\Delta(r) + \frac{3}{2} \ln\left(\frac{m^2}{\mu^2}\right) \right] \\ \Delta(r) &= -\frac{1}{2} + \int_0^1 dx (2 - x) \ln(r^2(1 - x) + x^2) \end{aligned} \quad (52)$$

with $r \equiv M/m$. There is also a one-loop QCD selfenergy diagram, to be added to the electroweak diagrams of fig. 4, which adds

$$2(\bar{a} - \bar{c}) = 2\frac{\alpha_s}{\pi}\left(\frac{4}{3} - \ln\left(\frac{m^2}{\mu^2}\right)\right) \quad (53)$$

Combining (51),(52),(53) gives [15]. See also [7].

$$\frac{m^*}{\bar{m}} = 1 + \frac{\alpha_s}{\pi}\left(\frac{4}{3} - \ln\left(\frac{m^2}{\mu^2}\right)\right) + \frac{\mathcal{Y}^2}{32\pi^2} \left[\Delta(r) + \frac{3}{2} \ln\left(\frac{m^2}{\mu^2}\right) \right] + \delta\bar{\zeta}_v \quad (54)$$

As in the bosonic case, the $\bar{M}S$ mass depends on the $\bar{M}S$ one-loop shift of the vev, $\delta\zeta_v$. (Again, as in the bosonic case, the $\delta\zeta_v$ in(54) can be absorbed in the definition of \bar{m} , which is generally gauge dependent when the gauge sector is included). The OS mass, m^* , which occurs in the OS version of (49) is the perturbative (all orders) pole mass - presumably closely related to the reported experimental top quark mass. Also as in the bosonic case, we see from (47) that Z_3 is fixed by the t mass renormalization condition (ζ_v being already fixed), so Z_3 is not available to define the top Yukawa coupling constant as the value of some trilinear vertex function. Thus \mathcal{Y}^* is fixed as

$$\mathcal{Y}^* = \sqrt{2} \frac{m^*}{v^*} \quad (55)$$

We relate the MOM and $\bar{M}S$ Yukawa coupling constants .

$$\frac{\mathcal{Y}^*}{\bar{\mathcal{Y}}} = \frac{m^* \bar{v}}{\bar{m} v^*} = \frac{m^*}{\bar{m}} \left[1 + \frac{1}{2}(\delta Z_\phi^* - \delta \bar{Z}_\phi) - \delta \bar{\zeta}_v \right] \quad (56)$$

Again, as for the bosonic coupling constant , the $-\delta\bar{\zeta}_v$ in the brackets is cancelled by the $\delta\bar{\zeta}_v$ from $\frac{m^*}{\bar{m}}$. With δZ_ϕ from (27),this becomes

$$\begin{aligned} \frac{\mathcal{Y}^*}{\bar{\mathcal{Y}}} = 1 &+ \frac{\alpha_s}{\pi}\left(\frac{4}{3} - \ln\left(\frac{m^2}{\mu^2}\right)\right) + \frac{\lambda}{32\pi^2} [-1] \\ &+ \frac{\mathcal{Y}^2}{32\pi^2} \left[-\frac{1}{2}N_c + \Delta(r) + (N_c + \frac{3}{2}) \ln\left(\frac{m^2}{\mu^2}\right) \right] \end{aligned} \quad (57)$$

Finally,

$$\lambda = \frac{\mathcal{Y}^2}{4} \frac{M^2}{m^2} \quad (58)$$

so the λ term in (57) may be replaced by $-r^2/4$ multiplied by $\mathcal{Y}^2/(32\pi^2)$ giving [15] and see also [7]

$$\begin{aligned} \frac{\mathcal{Y}^{2*}}{\bar{\mathcal{Y}}^2} = 1 &+ 2\frac{\alpha_s}{\pi} \left(\frac{4}{3} - \ln\left(\frac{m^2}{\mu^2}\right) \right) \\ &+ \frac{\mathcal{Y}^2}{32\pi^2} \left[-\frac{r^2}{2} + 2\Delta(r) - N_c + (2N_c + 3) \ln\left(\frac{m^2}{\mu^2}\right) \right] \end{aligned} \quad (59)$$

We comment on the $\delta\bar{\zeta}_v$ appearing in the equation relating m^* to \bar{m} (54). First note (15) that $\delta\bar{\zeta}_v \sim m^4/M^2 v^2 \sim \mathcal{Y}^4/\lambda$ can be sizable for a light Higgs boson. The top-quark two-point counter term (44) may be rewritten as

$$-i\Sigma_{ct} = i\delta Z_V(\gamma p - m) + i\delta Z_A \gamma p \gamma_5 - i\delta Z_m m \quad (60)$$

where

$$\begin{aligned} \delta Z_V &= \frac{1}{2}(\delta Z_L + \delta Z_t) , \\ \delta Z_A &= \frac{1}{2}(-\delta Z_L + \delta Z_t) , \\ \delta Z_m &= \delta Z_y + \frac{1}{2}\delta Z_\phi + \delta\zeta_v \end{aligned} \quad (61)$$

This is convenient for on-shell *MOM* renormalization, $\gamma p = m^*$ and $\delta\zeta_v^* = 0$. It is not suitable for $\bar{M}\bar{S}$. $\delta Z_m (\equiv \delta m/m)$ can not be treated as a $\bar{M}\bar{S}$ renormalization constant, pure Δ_ϵ , because $\delta\bar{\zeta}_v$ is nonzero and not Δ_ϵ . In fact, Z_m is not the renormalization of any parameter in the Lagrangian (2). There is no fermion bare mass m_0 . Thus changing Z_m is not equivalent to a simple reparametrization of the Lagrangian. The Lagrangian does contain a bare Yukawa coupling constant, renormalized by Z_y : $\mathcal{Y}^*/\bar{\mathcal{Y}} = \bar{Z}_y/Z_y^*$. So no $\delta\zeta_v$ appears in the equation relating \mathcal{Y}^* to $\bar{\mathcal{Y}}$ (57).

We conclude this section with an example showing the equivalence of a change of renormalization scheme and a reparametrization of the fields and parameters in the original Lagrangian. The example will be relevant to our discussion of the ρ parameter in the next section. Consider

$$\Delta\Sigma'_\phi = \Sigma'_{\phi_0}(0) - \Sigma'_{\phi_+}(0) \quad (62)$$

with the Σ'_ϕ given in (25),(26). In the *MOM* renormalization scheme we can write

$$\Delta\Sigma'^*_\phi = N_c y^{*2} \left[1 + y^{*2} (\rho_{FD}^{(2)}(r^*) + \rho_{ct}^{(2)*}(r^*)) + \dots \right] \quad (63)$$

with $y^2 \equiv \mathcal{Y}^2/(32\pi^2)$. The quantities $\rho^{(2)}$ are the results of a two-loop calculation [13],[14]. (The one-loop result follows from (25),(26), (27),(28)). The $\bar{M}\bar{S}$ version is

$$\Delta\bar{\Sigma}'_\phi = N_c \bar{y}^2 \left[1 + \bar{y}^2 (\rho_{FD}^{(2)}(\bar{r}) + \bar{\rho}_{ct}^{(2)}(\bar{r})) + \dots \right] \quad (64)$$

To the two-loop order calculated, the r^* and \bar{r} in the $\rho^{(2)}$ functions and the y^{*4} and \bar{y}^4 are equivalent. When we take the difference of the *MOM* and $\bar{M}\bar{S}$ functions, the $\rho_{FD}^{(2)}$ terms drop out.

$$\begin{aligned} \Delta\bar{\Sigma}'_\phi - \Delta\Sigma'^*_\phi &= N_c y^4 \left(\rho_{ct}^{(2)}(r) - \rho_{ct}^{(2)*}(r) \right) \\ &= N_c y^4 \left(2\Delta(r) + 3\ln\left(\frac{m^2}{\mu^2}\right) \right) \end{aligned} \quad (65)$$

The explicit values of the $\rho_{ct}^{(2)}$ used to obtain the second line are discussed in section four. $\Delta(r)$ is the function introduced in (52).

We now check that this result, the $\bar{M}S$ value of $\Delta\Sigma'_\phi$, is also obtained as a rescaling and reparametrization of $\Delta\Sigma'^*_\phi$. Since Σ is essentially the inverse of the two-point function, the required rescaling factor is \bar{Z}_ϕ/Z_ϕ^* :

$$\frac{\bar{Z}_\phi}{Z_\phi^*}(\Sigma'_{\phi_0^*}(0) - \Sigma'_{\phi_+^*}(0)) = (1 - Z_1 y^2) N_c \bar{y}^2 [1 + y^2 (Y_1 + \rho^{(2)*})] \quad (66)$$

where

$$\frac{Z_\phi^*}{\bar{Z}_\phi} = 1 + Z_1 y^2 = 1 + y^2 \left(-\frac{r^2}{2} - N_c + 2N_c \ln\left(\frac{m^2}{\mu^2}\right) \right) \quad (67)$$

$$\frac{y^{*2}}{\bar{y}^2} = 1 + Y_1 y^2 = 1 + y^2 \left(-\frac{r^2}{2} + 2\Delta(r) - N_c + (2N_c + 3) \ln\left(\frac{m^2}{\mu^2}\right) \right) \quad (68)$$

Combining these expressions gives the result (66) = (64)

2 Reduction formula

A renormalization scheme refers to perturbative calculations of τ -functions.

$$\tau = FT \langle T(\chi \dots) \rangle \quad (69)$$

The renormalization reparametrization (5) of the original Lagrangian (2) generates in perturbation theory ‘bare’ feynman diagrams (no counter terms, but renormalized masses and coupling constants), and counter terms (Feynman diagrams including δZ insertions).

$$\tau = \tau_{FD} + \tau_{ct} \quad (70)$$

The τ_{FD} are the same function of the renormalized masses and coupling constants in any renormalization schemes which introduces the same Z ’s (5). There is implicit scheme dependence in the definition of the renormalized masses and coupling constants (renormalization conditions). The explicit scheme dependence is in the δZ ’s in the τ_{ct} . To one-loop order, the τ_{ct} are just δZ times a polynomial in momenta. In higher orders, there are more complicated counter terms arising from the nesting of lower order counter terms in lower order Feynman diagrams.

To connect a τ -function to a physical observable (e.g. S-Matrix element) requires use of the LSZ reduction formulas. As a preliminary, we note that in all renormalization schemes the physical mass m^* is defined by the (perturbative) pole of the complete renormalized two-point function. (In this formal perturbative discussion we ignore difficulties with unstable and/or confined particles). The distinction is between schemes which fix the residue of the pole to be unity (*-schemes) and schemes which do not. The relation between the canonical (bare) field χ_0 and the various renormalized fields is

$$\chi_0 = \sqrt{Z^*} \chi^* = \sqrt{\bar{Z}} \bar{\chi} \quad (71)$$

The LSZ reduction formula is simple in a $*$ -scheme. (We show explicitly only one external line factor).

$$\mathcal{M} = \left(\lim_{q^2 \rightarrow m^{*2}} \right) (q^2 - m^{*2}) \tau^* = \underline{\tau}^* \quad (72)$$

where

$$\tau = D \underline{\tau} \quad (73)$$

D is the complete renormalized (e.g. D^* or \bar{D}) two-point function, and $\underline{\tau}$ is the fully amputated renormalized τ -function.

$$(q^2 - m^{*2}) \bar{D} \rightarrow \frac{Z^*}{\bar{Z}} \quad (74)$$

But from (69),(71) and from (73),(74)

$$\tau^* = \sqrt{\frac{\bar{Z}}{Z^*}} \bar{\tau}, \quad \underline{\tau}^* = \sqrt{\frac{Z^*}{\bar{Z}}} \bar{\tau} \quad (75)$$

Then

$$\mathcal{M} = \mathcal{M}^* = \underline{\tau}^* = \sqrt{\frac{Z^*}{\bar{Z}}} \bar{\tau} = \bar{\mathcal{M}} \quad (76)$$

τ^* is a function of the star renormalized masses and coupling constants ; $\bar{\tau}$ is a function of the bar renormalized masses and coupling constants .

3 Applications

3.1 $H \rightarrow f \bar{f}$ decay

The first application is to the decay of a heavy Higgs to fermion anti-fermion. Some of the contributing Feynman diagrams and counter terms are shown in fig5.

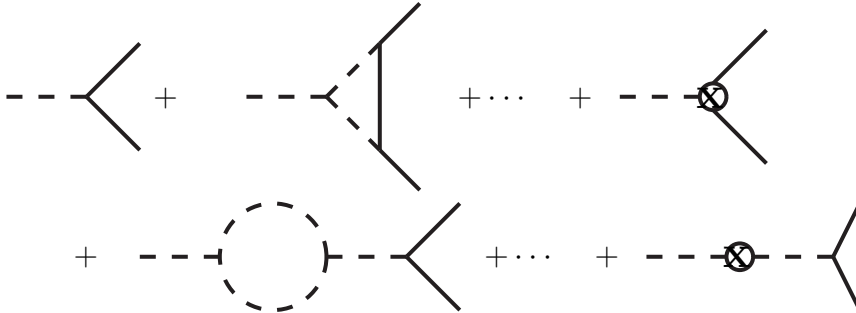


Fig.5. One-loop corrections to the amplitude for $H \rightarrow f\bar{f}$. In the heavy Higgs limit the boson loop diagram in the second line is dominant.

We consider the limit of a heavy Higgs, $M^2 \gg m^2$, $\lambda \gg \mathcal{Y}^2$. In our MOM scheme, with the Higgs field renormalized by the Goldstone boson field renormalization constant, Z_ϕ , and $\zeta_v = 1$, the proper vertex Feynman diagrams and counter term in the first line of fig5 are all of order $\lambda\mathcal{Y}^2$, or $\lambda\frac{m}{M}$ times the tree term ($\sim \mathcal{Y}$). The only source of pure λ one-loop corrections are the bosonic self-energy corrections to the external Higgs line. They survive LSZ amputation because $Z_\phi^* \neq Z_h^*$. (i.e. our *MOM* scheme is not a * -scheme with respect to the Higgs field). Thus in the heavy Higgs limit, the leading terms in \mathcal{M} are given by the external line factor times the tree term. The external line factor is determined by the analysis of the previous section (substitute Z_h^* for Z^* and Z_ϕ^* for \bar{Z}).

$$\begin{aligned}\mathcal{M} &\doteq \sqrt{\frac{Z_h^*}{Z_\phi^*}} \mathcal{M}_0^* = \mathcal{M}_0^* \left[1 + \frac{1}{2} (\delta Z_h^* - \delta Z_\phi^*) + \dots \right] \\ &= \mathcal{M}_0^* \left[1 + \frac{1}{2} \frac{\lambda^*}{16\pi^2} (13 - 2\sqrt{3}\pi) + \dots \right]\end{aligned}\quad (77)$$

The last result is from (34). This result was originally obtained by [16]. Recently the two-loop result has been calculated [4].

$$\Gamma = \Gamma_0^* \left\{ 1 + 2.12 \frac{\lambda^*}{16\pi^2} + \Gamma^{(2)*} \left(\frac{\lambda^*}{16\pi^2} \right)^2 + \dots \right\}, \quad \Gamma^{(2)*} = -32.66 \quad (78)$$

We can use (40), $\lambda^*(\bar{\lambda})$, to rewrite (78) in terms of the $\bar{M}\bar{S}$ coupling constant $\bar{\lambda}$. Setting the scale $\mu = M$, this gives

$$\Gamma = \Gamma_0^* \left\{ 1 + 2.12 \frac{\bar{\lambda}}{16\pi^2} - 51.05 \left(\frac{\bar{\lambda}}{16\pi^2} \right)^2 + \dots \right\} \quad (79)$$

which makes the apparent convergence problem worse.

Although (79) is a legitimate reparametrization of (78) (or (77)), it is not equivalent to a change of renormalization scheme from *MOM* to $\bar{M}\bar{S}$. (Γ_0^* contains \mathcal{Y}^{*2}) The amputated $\bar{M}\bar{S}$ τ -function, $\bar{\mathcal{I}}$, has no pure λ term. Suppressing fermion spinors,

$$\bar{\mathcal{I}}(\bar{\mathcal{Y}}, \bar{\lambda}) \doteq \frac{\bar{\mathcal{Y}}}{\sqrt{2}} + \bar{\lambda}(0 + \mathcal{O}(\mathcal{Y}^2)); \quad (80)$$

The $\bar{M}\bar{S}$ calculation of \mathcal{M} , from (76), (80), is

$$\bar{\mathcal{M}} = \sqrt{\frac{Z_h^*}{Z_h} \bar{\mathcal{I}}} \doteq \left\{ 1 + \frac{1}{2} (\delta Z_h^* - \delta \bar{Z}_h) + \dots \right\} \frac{\bar{\mathcal{Y}}}{\sqrt{2}} \quad (81)$$

The $\delta\bar{Z}_h$ just removes Δ_ϵ from δZ_h^* . So, with δZ_h^* from (33), this is

$$\bar{\mathcal{M}} = \sqrt{\frac{Z_h^*}{\bar{Z}_h}} \doteq \frac{\bar{\mathcal{Y}}}{\sqrt{2}} \left[1 + \frac{1}{2} \frac{\bar{\lambda}}{16\pi^2} (12 - 2\sqrt{3}\pi) + \dots \right] \quad (82)$$

We check that (82) is just the $\bar{M}\bar{S}$ reparametrization of \mathcal{M}^* , equation (77)

$$\mathcal{M}^* \doteq \frac{\mathcal{Y}^*}{\sqrt{2}} \left\{ 1 + \frac{1}{2} \frac{\lambda^*}{16\pi^2} (13 - 2\sqrt{3}\pi) + \dots \right\} \quad (83)$$

In the heavy Higgs limit considered,

$$\mathcal{Y}^* \doteq \bar{\mathcal{Y}} \left\{ 1 + \frac{\lambda}{16\pi^2} \left(-\frac{1}{2} + \dots\right) \right\} \quad (84)$$

Since the product $\mathcal{Y}\lambda$ is the same for star and bar to the order considered, this gives

$$\mathcal{M}^* \doteq \frac{\bar{\mathcal{Y}}}{\sqrt{2}} \left\{ 1 + \frac{1}{2} \frac{\bar{\lambda}}{16\pi^2} (12 - 2\sqrt{3}\pi) + \dots \right\} \doteq \bar{\mathcal{M}} \quad (85)$$

The decay rate calculated from $\bar{\mathcal{M}}$ is

$$\Gamma_{\bar{M}S} = \bar{\Gamma}_0 \left\{ 1 + 1.12 \frac{\bar{\lambda}}{16\pi^2} + (?) \left(\frac{\bar{\lambda}}{16\pi^2}\right)^2 + \dots \right\} \quad (86)$$

To complete the calculation of the coefficient of $\bar{\lambda}^2$ requires the two-loop λ^2 term in $\mathcal{Y}^*(\bar{\mathcal{Y}})$ which is not available at the time of writing. But already at one-loop level the convergence looks better.

3.2 ρ - parameter

Before moving on to the second application, we discuss anew the parameters in the reduced theory specified by the Lagrangian (2). The (bare) parameters in the Lagrangian are $\mu_0^2, \lambda_0, \mathcal{Y}_0$. After renormalization, (5), these are replaced by $\mu^2, \lambda, \mathcal{Y}$. The condition that the tree level vev of the shifted Higgs field vanish replaces $-\mu^2$ by λv^2 . In a MOM scheme, it is generally desirable to have the 'physical' masses as parameters, so we can trade λ, \mathcal{Y} for M, m via the relations $M^2 = 2\lambda v^2, m = \mathcal{Y} v/\sqrt{2}$, giving the set $\{v, M, m\}$. In $\bar{M}\bar{S}$ schemes, it may be advantageous to use the coupling constants, which are directly related to bare parameters in the Lagrangian, rather than the masses, which are inextricably bound up with the vev. Then observables are parametrized by the set $\{\bar{v}, \bar{\lambda}, \bar{\mathcal{Y}}\}$. In a MOM scheme, assuming that M^* (M_h) and m^* (m_t) are measured (to some accuracy), we still have to know v^* to give a numerical value for observables such as Γ_{MOM} (78). The determination of v^* requires us to go beyond the reduced ("gaugeless") theory we have been considering, and consider its embedding in the full standard Electroweak theory.

In the full theory, v^* is fixed by the relation

$$v^{*2} = 4 \frac{M_W^{*2}}{g^{*2}} \quad (87)$$

where M_W^* is the W mass, and $g^{*2} = e^{*2}/s_w^{*2}$

$$e^{*2} = 4\pi\alpha, \quad s_w^{*2} = 1 - M_W^{*2}/M_Z^{*2} \quad (88)$$

For precision electroweak calculations, since M_W is not so accurately known as α and M_Z , it is preferred to relate v^{*2} to the Fermi constant measured in μ decay.

$$\frac{1}{2v^{*2}} = \frac{g^{*2}}{8M_W^{*2}} = \frac{G_\mu}{\sqrt{2}}(1 - \Delta r^*) \quad (89)$$

Here Δr^* is the radiative correction to μ decay as originally computed by Marciano and Sirlin (See Hollik [5] for a review in the context of the complete one-loop *MOM* renormalization of the full electroweak theory by BSH [6]).

At one-loop order, Δr^* has quadratic dependence on m_t ($\sim g^2 m_t^2/M_W^2 \sim m_t^2/v^2$), but only logarithmic dependence on M_h (Veltman's screening theorem [8]), so in a calculation keeping only leading, quadratic in M_h , contributions, one can drop the Δr^* term in (89) and replace \mathcal{Y}^{*2} and λ^* in (78) by $2\sqrt{2}G_\mu m_t^{*2}$ and $G_\mu M_h^{*2}/\sqrt{2}$ respectively. The interesting feature of (78) is the quadratic and quartic dependence on M_h in one-loop and two-loop orders. Processes with an external Higgs line evade Veltman's screening theorem [8] that low energy processes with no external Higgs depend only logarithmically on the Higgs mass in one-loop order. For finite M_h , an accurate calculation requires additional contributions to (78), some of which we have noted. (See fig. 5. See also [1]).

The second application involves the ρ parameter. There are several definitions of ρ , all of which are unity at tree level in the standard electroweak theory.[11] There are also different definitions of $\Delta\rho$, all of which provide a measure of the violation of the electroweak i-spin symmetry produced by the unequal fermion Yukawa coupling constants. The definition adopted by [13],[14] is that ρ is the ratio of the effective low energy neutral current to charged current four-Fermi coupling constants. The effective low energy charged current four-Fermi coupling constant is the Fermi constant measured in μ decay (89). The effective low energy neutral four-fermi coupling constant is normalized by some function of the Weinberg angle such that the ratio is unity at tree level. Thus one could extract ρ from the ratio of the low energy cross sections for $\nu_\mu e$ to $\nu_\mu e$ (Z -exchange) and $\nu_\mu e$ to $\mu \nu_e$ (W -exchange); see fig. 6.

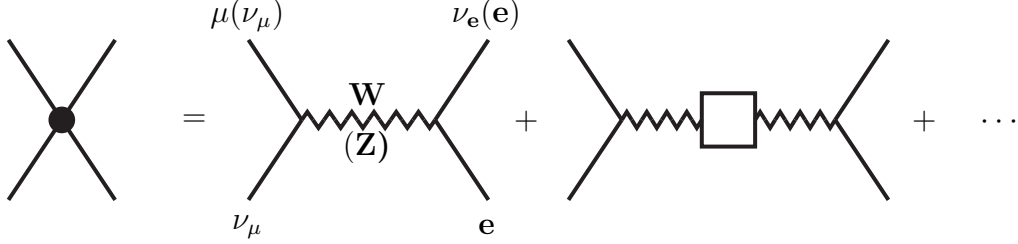


Fig.6. Radiative corrections to the ρ -ratio in the "gaugeless" limit $m_H, m_t \gg m_W$.

One can choose a renormalization scheme (either MOM or \bar{MS}) in which the only source of contributions linear in m_t^2 or M_h^2 is selfenergy corrections to the exchanged vector boson propagator (fig. 6). Then

$$\rho \doteq \frac{1}{c_w^2} \frac{D_Z(0)}{D_W(0)} = \frac{1 - \Pi_W(0)/M_W^2}{1 - \Pi_Z(0)/M_Z^2} \quad (90)$$

Here, D, Π, M are all renormalized quantities, in whatever scheme, not yet specified. Again following [13],[14], define

$$\Delta\rho = 1 - \frac{1}{\rho} \doteq \frac{P_Z - P_W}{1 - P_W} \quad (91)$$

We have defined

$$P_{Z,W} \equiv \Pi_{Z,W}(0)/M_{Z,W}^2$$

The Feynman diagrams for the radiative corrections to μ decay, Δr in (89), are crossed versions of those for W exchange in fig. 6. Quadratic in m_t contributions to Δr come from W selfenergy insertions.

$$\Delta r \doteq \frac{\Pi_w(0)}{M_W^2} \quad (92)$$

Then, with the definition of $\Delta\rho$ (91),

$$\Delta\rho \doteq \frac{1}{1 - \Delta r} (P_Z - P_W) \quad (93)$$

The gauge vector boson selfenergy functions $\Pi_{W,Z}$ are not included in the reduced ("gaugeless") theory we have considered. But in the full theory there are Slavnov-Taylor-Ward identities which relate the unrenormalized Π, Σ , for both W and Z.

$$\frac{\Pi_V(0)}{M_V^2} = \frac{\Sigma_\phi(k^2)}{k^2} \Big|_{k^2=0}, \quad V=W,Z \quad \phi = \phi_+, \phi_0 \quad (94)$$

The boldface Π, Σ are given by the sum of (regularized) Feynman diagrams with no counter terms. In the detailed renormalization scheme of [6], the γ, Z mixing lead to renormalized Π functions which have explicit dependence on s_w (sine of the Weinberg angle), which the renormalized Σ functions (25),(26) do not have. As already mentioned below (27),(28) when the full electroweak theory is considered, one may prefer to fix δZ by a different condition than (27). A choice which leads to renormalized Σ functions which satisfy S-T-W identities is given in App C. This does not have to be discussed here because the additional terms, dependent on the Weinberg angle, drop out in the difference between the charged and neutral functions (for those terms which could contribute quadratic in m_t or M_h)

$$\begin{aligned} \frac{\Pi_Z(0)}{M_Z^2} - \frac{\Pi_W(0)}{M_W^2} &\doteq \frac{\Pi_Z(0)}{M_Z^2} - \frac{\Pi_W(0)}{M_W^2} = \Sigma'_{\phi_0}(0) - \Sigma'_{\phi_+}(0) \\ &= \Sigma'_{\phi_0}(0) + \delta Z_\phi - \Sigma'_{\phi_+}(0) - \delta Z_\phi = \Sigma'_{\phi_0}(0) - \Sigma'_{\phi_+}(0) \end{aligned} \quad (95)$$

These equations with (28),(27) give the one-loop result

$$\Delta\rho \doteq \Sigma'_{\phi_0}(0) - \Sigma'_{\phi_+}(0) = \delta Z_{\phi_0} - \delta Z_{\phi_+} = N_c \frac{\mathcal{Y}^2}{32\pi^2} \quad (96)$$

Two groups [13],[14] have computed the two-loop electroweak contributions to $\Delta\rho$ in *MOM*, using the STW relation (95). (The authors of the second reference have also directly calculated with the vector boson selfenergies and obtained the same result, thus verifying (95) through two-loop order). The QCD correction to the one-loop electroweak result has been calculated in [12]

$$\Sigma'_{\phi_0}(0) - \Sigma'_{\phi_+}(0) = N_c \frac{\mathcal{Y}^{*2}}{32\pi^2} \left\{ 1 - \frac{2}{9} \frac{\alpha_s}{\pi} (\pi^2 + 3) + \frac{\mathcal{Y}^{*2}}{32\pi^2} \rho^{(2)*}(r) \right\} \quad (97)$$

$$\Delta\rho = \frac{1}{1 - \Delta r^*} N_c \frac{\mathcal{Y}^{*2}}{32\pi^2} \left\{ 1 - \frac{2}{9} \frac{\alpha_s}{\pi} (\pi^2 + 3) + \frac{\mathcal{Y}^{*2}}{32\pi^2} \rho^{(2)*}(r) \right\} \quad (98)$$

By (55), (89),

$$\frac{\mathcal{Y}^{*2}}{1 - \Delta r^*} = 2\sqrt{2} G_\mu m^{*2} \quad (99)$$

which is the motivation for the definition of $\Delta\rho$ in (91). From (93),(95),(97),(99), we arrive at

$$\begin{aligned} \Delta\rho &= N_c x_t^* \left\{ 1 - \frac{2}{9} \frac{\alpha_s}{\pi} (\pi^2 + 3) + x_t^* \rho^{(2)*}(r) + \dots \right\}, \\ x_t^* &\equiv \frac{G_\mu m_t^{*2}}{8\sqrt{2}\pi^2}. \end{aligned} \quad (100)$$

All of the electroweak quantities in this formula are computed in MOM, so this $\Delta\rho$ is $\Delta\rho_{MOM}$. The small r and large r behaviors of $\rho^{(2)*}(r)$ are [13], [14]

$$\rho^{(2)*}(r) \simeq -2\pi^2 + 19 \quad (r \rightarrow 0) \quad (101)$$

$$\rho^{(2)*}(r) \simeq \frac{3}{2}(\ln r^2)^2 - \frac{27}{2}\ln r^2 + \pi^2 + \frac{49}{4} \quad (r \gg 1) \quad (102)$$

So for the *MOM* calculation of $\Delta\rho$, the large m_t behavior is of order $G_\mu m_t^2$ and $(G_\mu m_t^2)^2$, and the large M_h behavior is order $O((G_\mu m_t^2)^2(\ln M_h^2/m_t^2)^2)$.

We now consider some reparametrizations of this result. First, we collect the transformation equations for the relevant parameters. Copying (54),(57)

$$\frac{m^*}{\bar{m}} = 1 + \frac{\alpha_s}{\pi} \left(\frac{4}{3} - \ln\left(\frac{m^2}{\mu^2}\right) \right) + \frac{\mathcal{Y}^2}{32\pi^2} \left[\Delta(r) + \frac{3}{2} \ln\left(\frac{m^2}{\mu^2}\right) \right] + \delta\bar{\zeta}_v \quad (103)$$

$$\begin{aligned} \frac{\mathcal{Y}^*}{\bar{\mathcal{Y}}} &= 1 + \frac{\alpha_s}{\pi} \left(\frac{4}{3} - \ln\left(\frac{m^2}{\mu^2}\right) \right) + \frac{\lambda}{32\pi^2} [-1] \\ &+ \frac{\mathcal{Y}^2}{32\pi^2} \left[-\frac{1}{2}N_c + \Delta(r) + \left(N_c + \frac{3}{2}\right) \ln\left(\frac{m^2}{\mu^2}\right) \right] \end{aligned} \quad (104)$$

and, see (40)

$$\begin{aligned} \frac{v^*}{\bar{v}} &= 1 + \frac{1}{2}(\delta\bar{Z}_\phi - \delta Z_\phi^*) + \delta\bar{\zeta}_v \\ &= 1 + \frac{\lambda}{32\pi^2} [1] + N_c \frac{\mathcal{Y}^2}{32\pi^2} \left[-\ln\frac{m^2}{\mu^2} + \frac{1}{2} \right] + \delta\bar{\zeta}_v \end{aligned} \quad (105)$$

The first reparametrization is just to transform from "on-shell" top mass m_t^* to the \overline{MS} mass in (100). This produces

$$\begin{aligned} \Delta\rho &= N_c \bar{x}_t \left\{ 1 - \frac{2}{9} \frac{\alpha_s}{\pi} \left[\pi^2 - 9 + 9 \ln\left(\frac{m^2}{\mu^2}\right) \right] \right. \\ &\quad \left. + \bar{x}_t \left[\rho^{(2)*}(r) + 2\Delta(r) + 3 \ln\left(\frac{m^2}{\mu^2}\right) \right] + 2\delta\bar{\zeta}_v \right\} \end{aligned} \quad (106)$$

$$\bar{x}_t \equiv \frac{G_\mu \bar{m}_t^2}{8\sqrt{2}\pi^2} \quad (107)$$

To two-loop order, all the quantities (m , $r = M/m$) may be taken to be \overline{MS} quantities. Thus this $\Delta\rho$ is a candidate for $\Delta\rho_{\overline{MS}}$. As discussed at the end of section two, reparametrization of the quark mass in the electroweak theory is questionable because there is no bare mass parameter in the Lagrangian; and it leads to the appearance of the singular $\delta\bar{\zeta}_v$ in (106). This suggests that a better reparametrization would be of

the Yukawa coupling constant .[15] There is a bare Yukawa coupling constant in the Lagrangian and the transformation from \mathcal{Y}^* to $\bar{\mathcal{Y}}$ (104) does not involve $\delta\bar{\zeta}_v$. Substitution of (104) into (98) yields

$$\begin{aligned}\Delta\rho &= \frac{1}{1-\Delta r^*} N_c \bar{y}^2 \left\{ 1 - \frac{2}{9} \frac{\alpha_s}{\pi} \left[\pi^2 - 9 + 9 \ln\left(\frac{m^2}{\mu^2}\right) \right] \right. \\ &\quad \left. + \bar{y}^2 \left[\rho^{(2)*} - \frac{r^2}{2} + 2\Delta(r) - N_c + (2N_c + 3) \ln\left(\frac{m^2}{\mu^2}\right) \right] \right\} \\ y^2 &= \frac{\mathcal{Y}^2}{32\pi^2}\end{aligned}$$

There are two very interesting features of this parametrization. If we set the scale mass μ equal to m_t , the coefficient of α_s becomes very small [9] and the two-loop electroweak correction to the one-loop result becomes larger than the QCD correction. The second is the appearance of the $-r^2/2$, which comes from the scalar coupling constant contribution to \mathcal{Y}^* ($\bar{\mathcal{Y}}$), (57). Thus the large M_h behavior in this parametrization is order $Gm_t^2 GM_h^2$.

However, the presence of the factor $1/(1-\Delta r^*)$ implies that this is a parametrization in a mixed scheme. In appendix A we show that

$$\frac{1}{1-\Delta r} \frac{1}{\bar{v}^2} \doteq \frac{1}{1-\Delta r^*} \frac{1}{v^{*2}} = \sqrt{2} G_\mu \quad (109)$$

From which follows

$$\frac{1}{1-\Delta r^*} = \frac{v^{*2}}{\bar{v}^2} \frac{1}{1-\Delta \bar{r}} = \frac{1}{1-\Delta \bar{r}} \left\{ 1 + N_c y^2 \left[\frac{1}{2} r^2 + N_c (1 - 2 \ln(\frac{m^2}{\mu^2})) \right] + 2\delta\bar{\zeta}_v \right\} \quad (110)$$

Substitution of (110) into (108) yields

$$\begin{aligned}\Delta\rho &= \frac{1}{1-\Delta \bar{r}} N_c \bar{y}^2 \left\{ 1 - \frac{2}{9} \frac{\alpha_s}{\pi} \left[\pi^2 - 9 + 9 \ln\left(\frac{m^2}{\mu^2}\right) \right] \right. \\ &\quad \left. + \bar{y}^2 \left[\rho^{(2)*} + 2\Delta(r) + 3 \ln\left(\frac{m^2}{\mu^2}\right) \right] + 2 \delta\bar{\zeta}_v \right\} \quad (111)\end{aligned}$$

But from (64)

$$\rho^{(2)*} + 2 \Delta(r) + 3 \ln\left(\frac{m^2}{\mu^2}\right) = \bar{\rho}^2 \quad (112)$$

and by (109)

$$\frac{\bar{y}^2}{1-\Delta \bar{r}} = \bar{x}_t \quad (113)$$

With these relations, this $\Delta\rho$ from (111) is identical to $\Delta\rho_{\overline{MS}}$ from (106).

These alternative parametrizations of $\Delta\rho$ may be used as one estimate of the error arising from truncation of the perturbation series at finite (in this case, two-loop) order. Since these are just reparametrizations, exact (all orders) calculations, using exact (all

orders) relations between the parameters, must give the same numerical result. When the exact result for two different parametrizations is separated into a finite order calculated part plus uncalculated remainder, it follows that at least one of the remainders is the same order of magnitude as the difference of the two finite order calculated terms.

Because of the explicit $\ln \mu^2$ dependence in the formulas (103) to (112), and the implicit dependence on $\ln \mu^2$ in choice of value for α_s which appears in these formulas, the questions of truncation error and scheme dependence become entangled with questions of scale dependence. Thus there are many choices to make as to quantities to compare. We have chosen to compare $\Delta\rho_{MOM}$ (100), and the alternative parametrizations we have called $\Delta\rho_{mix}$ (108), and $\Delta\rho_{\overline{MS}}$ (106), or (111). And we make these comparisons for three choices of scale(s). First, we take all explicit μ s equal to M_W , but α_s at scale of m_t . Second, we take all μ s which come from QCD to be m_t , and all μ s from weak interactions to be M_W . Third, we take all μ s equal to m_t . The results are given in Tables 1,2,3.

Table 1. Two-loop values of $\Delta\rho$ in different parametrizations.

$\mu = M_W$. $m_t = 180$. $r = M_h/180$. $\alpha_s(m_t) = .107$.

| M_h | $10^3 \Delta\rho$ | | | | |
|-------|-------------------|------|------|-----------------|-------|
| | MOM | MIX | diff | \overline{MS} | diff |
| 60 | 9.05 | 9.07 | -.02 | 7.76 | 1.29 |
| 150 | 8.96 | 8.99 | -.03 | 9.00 | -.04 |
| 300 | 8.87 | 8.91 | -.04 | 8.82 | .05 |
| 600 | 8.80 | 8.84 | -.04 | 7.36 | 1.44 |
| 1000 | 8.77 | 8.78 | -.01 | -8.09 | 16.86 |

Table 2. Two-loop values of $\Delta\rho$ in different parametrizations.

$\mu_{QCD} = m_t$. $\mu_w = M_W$. $m_t = 180$.

| M_h | $10^3 \Delta\rho$ | | | | |
|-------|-------------------|------|------|-----------------|-------|
| | MOM | MIX | diff | \overline{MS} | diff |
| 60 | 9.05 | 9.06 | -.01 | 7.27 | 1.78 |
| 150 | 8.96 | 8.97 | -.03 | 8.91 | .05 |
| 300 | 8.87 | 8.89 | -.02 | 8.92 | -.05 |
| 600 | 8.80 | 8.84 | -.04 | 8.28 | .52 |
| 1000 | 8.77 | 8.86 | -.09 | -2.26 | 11.03 |

Table 3. Two-loop values of $\Delta\rho$ in different parametrizations.
 $\mu = m_t$. $m_t = 180$.

| M_h | $10^3 \Delta\rho$ | | | | |
|-------|-------------------|------|------|-----------------|------|
| | MOM | MIX | diff | \overline{MS} | diff |
| 60 | 9.05 | 9.12 | -.07 | 4.60 | 4.45 |
| 150 | 8.96 | 9.04 | -.08 | 8.94 | .02 |
| 300 | 8.87 | 8.97 | -.10 | 8.93 | -.06 |
| 600 | 8.80 | 8.90 | -.10 | 8.71 | .09 |
| 1000 | 8.77 | 8.88 | -.11 | 4.28 | 4.49 |

From the tables we see that for values of $r = M_h/m_t$ departing significantly from one, at least one of the truncated perturbation series for $\Delta\rho$ is very bad. The 'visible' problem is coming from the $\delta \bar{\zeta}_v$ which appears in the parametrization we have called $\Delta\rho_{\overline{MS}}$ (106), (111), and which blows up as $1/r^2$ or as r^2 as r goes to zero or infinity. This supports the contention that it is better to transform the Yukawa coupling constant than the quark mass. Of course we should not be surprised that the perturbation theory has failed for a Higgs mass of order one TeV. It has been long known that partial wave unitarity is violated by the tree level perturbative amplitudes for a Higgs of this mass. In fact, despite the small differences between the truncated perturbative results for $\Delta\rho_{MOM}$ and $\Delta\rho_{MIX}$, we should expect both of these perturbative expansions to be bad for a Higgs mass of one TeV. (The smallness of the difference of the truncated expansions is a necessary, but not sufficient, condition for both of the expansions to be good). All we can say is that for Higgs mass in the range of one hundred to six hundred GeV, these results are consistent with the two-loop results for $\Delta\rho_{MOM}$ and $\Delta\rho_{MIX}$ being accurate to the order of one percent.

For an alternative and more detailed discussion, with perhaps some difference of interpretation, we refer to Kniehl and Sirlin [17].

4 Appendix A

In this appendix we neglect numerical factors $32\pi^2, \sqrt{2}$ (e.g. absorb them in definitions of \mathcal{Y} and G .) Then (99) is written as

$$\frac{1}{1 - \Delta r^*} \mathcal{Y}^{*2} = G_\mu m^{*2} \quad (114)$$

and (92) is

$$\Delta r \doteq \frac{\Pi_W(0)}{M_W^2} \quad (115)$$

Then

$$M_W^{*2} (1 - \Delta r^*) \doteq M_W^{*2} - \Pi_W^{*2}(0) \quad (116)$$

(Suppress the subscript W)

$$\Pi(k^2) = \mathbf{\Pi}(k^2) + \delta M^2 + (k^2 - M^2) \delta Z \quad (117)$$

in either MOM or $\bar{M}S$. Then

$$\delta M^{*2} = -\mathbf{\Pi}(M^{*2}) \quad (118)$$

$$\begin{aligned} 0 &= M^{*2} - \bar{M}^2 + \bar{\Pi}(M^{*2}) \\ \bar{\Pi}(M^{*2}) - \bar{\Pi}(0) &= \mathbf{\Pi}(M^{*2}) + M^{*2} \delta \bar{Z} - \mathbf{\Pi}(0) \\ &= -\Pi^*(0) + (\delta \bar{Z} - \delta Z^*) M^{*2} \end{aligned} \quad (119)$$

Then

$$\begin{aligned} \bar{M}^2 (1 - \overline{\Delta r}) &= \bar{M}^2 - \bar{\Pi}(0) = M^{*2} + \bar{\Pi}(M^{*2}) - \bar{\Pi}(0) \\ &= M^{*2} - \Pi^*(0) + (\delta \bar{Z} - \delta Z^*) M^{*2} \end{aligned} \quad (120)$$

Then

$$\begin{aligned} \frac{\bar{g}^2}{\bar{M}_W^2} \frac{1}{1 - \overline{\Delta r}} &= \frac{\bar{g}^2}{\bar{M}_W^2 - \bar{\Pi}_W(0)} = \frac{g^{*2} (1 + 2 (\delta Z_g^* - \delta \bar{Z}_g))}{M_W^{*2} (1 - \Pi_W^*(0)/M_W^{*2} - (\delta Z_W^* - \delta \bar{Z}_W))} \\ &= \frac{g^{*2}}{M_W^{*2}} \frac{1}{1 - \Delta r^*} [1 + 2 (\delta Z_g^* - \delta \bar{Z}_g) + \delta Z_W^* - \delta \bar{Z}_W + \text{two-loop}] \end{aligned} \quad (121)$$

The combination of renormalization constants which appears here is precisely the combination which enters into the renormalization of the proper vertex function. ($2(\delta Z_1^W - \delta Z_2^W)$) in the notation of eq. (4.10) of [5]. In

the *MOM* scheme of BSH [6], this combination has no contribution linear in m_t^2 , so we can write

$$\frac{1}{1 - \overline{\Delta r}} \frac{1}{\bar{v}^2} \doteq \frac{1}{1 - \Delta r^*} \frac{1}{v^{*2}} \quad (122)$$

5 Appendix B

At the end of section one (see 65) we made use of the fact that the difference of the two-loop MOM and $\bar{M}S$ contributions to $\Delta\rho$ is just equal to the difference of the corresponding two-loop counter terms .

$$\bar{\rho}^{(2)} - \rho^{*(2)} = \bar{\rho}_{ct}^{(2)} - \rho_{ct}^{*(2)} \quad (123)$$

We have independently computed these counter terms .

$$\rho_{ct}^{*(2)} = 3\Delta_\epsilon - 3\ln\left(\frac{m^2}{\mu^2}\right) - 2\Delta(r) - \frac{3}{2}, \quad \bar{\rho}_{ct}^{(2)} = 3\Delta_\epsilon - \frac{3}{2} \quad (124)$$

One can readily verify that $\rho_{ct}^{*(2)}$ given here (see (52)) is identical to eq (11) of the second FTJ paper [14]. However FTJ have omitted the $-3/2$ from the $\bar{M}S$ counter term . But

the origin of this term is clear. It is the product of $1/\epsilon$ from one-loop δZ times a term of order ϵ from the dimensionally regulated one-loop integral in which the δZ counter term is embedded. Since the divergent part of δZ and the "bare" one-loop dimensionally regulated Feynman integrals are the same for MOM and $\bar{M}S$, the same $-3/2$ occurs in both counter terms. It is only at one-loop order, where the counter term is pure δZ , that the $\bar{M}S$ counter term only subtracts $1/\epsilon$ from the Feynman integrals. Finally, note that the example given at the end of section two verifies the difference of these two counter terms. (This point has also been commented on by Kniehl and Sirlin [17]).

6 Appendix C

For the full Electroweak theory we follow generally the renormalization prescriptions of [6],[5]. In the vector boson sector, they impose five renormalization conditions : the renormalized M_W, M_Z are the physical masses. The photon mass is zero, and the residue of the pole of the renormalized photon two-point function is unity. The mixed $\gamma-Z$ two-point function is zero at $k^2 = 0$. There are only four renormalization constants: $Z_W, Z_B, Z_g, Z_{g'}$. So Z_ϕ is also used to enforce these conditions. The resulting δZ_ϕ , proportional to the top Yukawa coupling constant squared, is

$$\delta Z_\phi \doteq x \left[-\Delta_\epsilon + \ln \frac{m^2}{\mu^2} + \frac{1}{2} \left(\frac{c^2}{s^2} - 1 \right) \right] \quad (125)$$

$$x = N_c \frac{\mathcal{Y}^2}{16\pi^2} \quad (126)$$

Substituting this δZ_ϕ into (25),(26), we find for the terms proportional to x,

$$\Sigma'_{\phi_+} \doteq -x \left[\frac{1}{2} \frac{c^2}{s^2} \right] \quad \Sigma'_{\phi_0} \doteq -x \left[\frac{1}{2} \frac{c^2 - s^2}{s^2} \right] \quad (127)$$

The terms proportional to the top mass squared in the vector boson selfenergy functions are

$$\frac{\Pi_W(0)}{M_W^2} \doteq N_c \frac{g^2}{16\pi^2} \frac{m^2}{M_W^2} \left[\frac{-1}{4} \frac{c^2}{s^2} \right], \quad \frac{\Pi_Z(0)}{M_Z^2} \doteq N_c \frac{g^2}{16\pi^2} \frac{m^2}{M_Z^2} \left[\frac{-1}{4} \frac{c^2 - s^2}{s^2} \right] \quad (128)$$

Using

$$\mathcal{Y}^2 = g^2 \frac{m^2}{2 M_W^2}, \quad M_Z^2 c^2 = M_W^2$$

we see that the S-T-W relations are satisfied by both the charged and neutral functions and the differences are independent of the Weinberg angle.

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